

①  $X, Y$  independent,  $X$  has a density, does  $X+Y$  also have a density?

②  $0 \leq X \leq 1$ . For what distributions of  $X$  does  $\text{Var} X$  have the largest value?

③  $X, Y$  i.i.d.  $N(0, 1)$ . What is the distribution of  $\left( \frac{X_1}{\sqrt{X_1^2 + X_2^2}}, \frac{X_2}{\sqrt{X_1^2 + X_2^2}} \right)$ ?

④  $P(N=i) = \frac{1}{3}, i=1, 2, 3$ .  
 $X_1, X_2, X_3$  i.i.d.  $N(0, 1)$  independent of  $N$ .  
 $X = \sum_{i=1}^N X_i$ . Is  $X$  also normal?

Probability and  
Statistics.